Book of abstracts

A priori estimates for quasilinear elliptic problems

Laura Baldelli - Imag, Universidad de Granada (Spain)

Abstract

In this talk, we will focus on a priori estimates of the type

$$u(x) + |Du(x)|^{\alpha_1} \leq C(1 + \text{dist}^{-\alpha_2}(x, \partial\Omega)), \ x \in \Omega \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is an arbitrary domain, $\alpha_i > 0$ for $i = 1, 2$, for any u nonnegative solutions of a large class of elliptic quasilinear equations and systems involving the p-Laplacian operator on arbitrary domains of $\mathbb{R}^N$ and a convective term in the reaction. Estimates of the type (1) are those that Serrin and Zou in [Acta Math., (2002)] call universal a priori estimates, because they are independent of the solutions and do not need any boundary conditions.

Our main theorems, new even for the Laplacian operator, extend previous estimates by Poláčik, Quitter and Souplet in [Duke Math. J., 2007] to very general nonlinearities admitting solely a lower bound, yielding a curious dichotomy. The technique used it is based on rescaling arguments combined with a key "doubling" property, which is different from the celebrated blow-up technique due to Gidas and Spruck.

Singular Liouville equations in high and low dimensions

Gabriele Mancini - Università degli Studi di Bari Aldo Moro (Italy)

Abstract

In this talk, I will discuss existence, classification, and non-degeneracy results for solutions to singular Liouville-type equations in dimension one. This problem has applications in the mathematical modeling of galvanic corrosion phenomena for ideal electrochemical cells consisting of an electrolyte solution confined in a bounded domain with an electrochemically active portion of the boundary. In higher dimensions, Liouville equations have applications to prescribed curvature problems in conformal geometry, where solutions correspond to constant Q-curvature metrics on Euclidean space, with a singular point at the origin. After providing a general overview of the existing literature, I will focus on the one-dimensional case and prove that solutions are non-degenerate, under mild assumptions on the singular weight. The proof relies on the use of harmonic extensions and conformal transformations to rewrite the linearized Liouville equation as a Steklov eigenvalue problem on either the intersection or the union of two disks. These results were obtained in collaboration with A. DelaTorre and A. Pistoia.

Concentration and oscillation analysis of semilinear elliptic equations with exponential growth in a disc

Daisuke Naimen - Muroran Institute of Technology (Japan)

Abstract

We study infinite concentration and oscillation phenomena on blow-up positive solutions of semilinear elliptic equations with supercritical exponential growth. We first detect infinite sequences of bubbles by a scaling technique. The precise description of each bubble is completed via a limit equation with suitable energy recurrence formulas. Thanks to this, we arrive at a key observation, the infinite sequences of bubbles cause infinite oscillations, around singular solutions, of blow-up solutions. This leads to a proof of infinite oscillations of bifurcation diagrams which yield the existence of infinitely many solutions.

Existence of time-periodic solutions to Maxwell's equations

Sebastian Ohrem - Karlsruhe Institut für Technologie (Germany)

Abstract

We consider Maxwell's equations without charges and currents

$$\nabla \cdot \mathbf{D} = 0, \qquad \nabla \times \mathbf{E} = -\mathbf{B}_t,$$
$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{H} = \mathbf{D}_t.$$

For material relations

$$\mathbf{B} = \mu_0 \mathbf{H}, \qquad \mathbf{D} = \mathbf{D}(\mathbf{E}),$$

modeling Kerr-type optical materials, with carefully chosen inhomogeneous material coefficients, we show existence of time-periodic, real-valued, traveling, localized solutions, called breathers. We use variational methods. This is joint work with Wolfgang Reichel (KIT)

On a pure Neumann Lane-Emden system: existence, convergence, and related results

Delia Schiera - Instituto Superior Técnico, Universidade de Lisboa (Portugal)

Abstract

We will consider a Lane-Emden system on a bounded regular domain with Neumann boundary conditions and (sub)critical nonlinearities. In the critical regime, we show that, under suitable conditions on the exponents in the nonlinearities, least-energy (sign-changing) solutions exist. Moreover, through a suitable nonlinear eigenvalue problem, we prove convergence of solutions in dependence of the exponents of the nonlinearities in the (sub)critical range. Finally, I will briefly discuss related results on multiplicity, symmetry breaking, and regularity. Based on joint works with A. Pistoia, A. Saldaña and H. Tavares.

Stability, robustness and regularity for the Biot's equations in poroelasticity

Pietro Zanotti - Università degli Studi di Milano (Italy)

Abstract

Poroelasticity models the coupling between mechanical deformation and fluid flow in porous media. The Biot equations are one of the most popular poroelastic models and their approximation has attracted a growing attention in recent years. The derivation of sharp and robust error estimates for an approximation method calls for two-sided stability and regularity estimates of the exact solution. In other words, suitable norms are sought so that the weak formulation of the equations establishes an isomorphism between the solution and the data spaces. The talk reviews the existing results and introduces some new developments and open questions in this direction. This is a joint work with C. Kreuzer (TU Dortmund).