Riemann International School of Mathematics

RISM

INdAM

CONGRESS

10-14 June 2024

Mean field models in optimal control and multi-agent dynamics

Villa Toeplitz, Varese

BOOKLET

Event schedule and Presentations abstracts

SUPPORTED BY

Università degli Studi di Padova

Dipartimento Matematica

RISM.IT

Villa Toeplitz
Via G.B. Vico, 46
21100 Varese VA

Summary

Event schedule

10/06/2024

Time	Title	Speaker
09:15	Registration / Opening	-
10:00	On HJB equations in P or in M	P.-L. Lions
10:45	Coffee Break	-
11:15	Long-time behaviour of Keller-Segel type systems modeling local sensing	A. Trescases
12:00	Ergodic behaviour of deterministic Mean Field Games with non-monotone interactions	M. Bardi
12:45	Lunch	-
14:30	Hidden monotonicity and canonical transformations for Mean Field Games and master equations	A. Meszaros
15:15	A mean-field-game approach to overfishing	Z. Kobeissi
16:00	Coffee Break	-
16:30	A priori estimates and large population limits for some nonsymmetric Nash systems with semimonotonicity	D. Redaelli
16:55	From entropic propagation of chaos to exponential concentration bounds for stochastic particle systems	J. Jackson

11/06/2024

Time	Title	Speaker
09:15	A free boundary problem arising in mean field games	P. Cardaliaguet
10:00	On mean field games and mean field control in infinite dimension	F. Gozzi
10:45	Coffee Break	-
11:15	A MFG model of knowledge diffusion: convergence to a balanced growth path	L. Rossi
12:00	Filling the gap between Hamilton-Jacobi equations and stochastic individual based models	S. Mirrahimi
12:45	Lunch	-
14:30	On quasi-stationary Mean Field Games of Controls	F. Camilli
15:15	Cluster Formation in Iterated Mean Field Games	E. Matter
15:40	Stable Optimal Parameters of Mean-Field Neural ODEs	S. Daudin
16:00	Coffee Break	-
16:30	Some results on boundary value problems for Choquard equations	C. Bernardini
16:55	A single player and a mass of agents: a pursuit evasion-like game	L. Marzufero

12/06/2024

Time	Title	Speaker
09:15	Dissipative evolutions of probability measures	G. Savaré
10:00	Deterministic many particle limits for degenerate second order traffic models	E. Radici
10:45	Coffee Break	-
11:15	Remarks on Potential Mean Field Games	J. Graber
12:00	Optimal control of conditional processes: old and new	R. Carmona

13/06/2024

Time	Title	Speaker
09:15	Synchronization games	H.M. Soner
10:00	Deterministic Mean Field Games with non coercive Hamiltonian.	P. Mannucci
10:45	Coffee Break	-
11:15	Weak-strong uniqueness for mean-field games	D. Gomes
12:00	Some well-posedness results for HJB equations on the space of probability measures	B. Seeger
12:45	Lunch	-
14:30	Parabolic first-order Mean Field Games with Neumann boundary conditions	M. Ricciardi
14:55	On the turnpike property in mean field games	K. Eichinger
15:20	Introduction to first order MFG under state constraints	R. Capuani
15:45	Sharp convergence rates for mean field control in the region of strong regularity	N.Mimikos-Stamatopoulos
16:00	Coffee break	-
16:15	End of session	

14/06/2024

Time	Title	Speaker
09:15	Miscellaneous on Hamilton Jacobi equations on the space of probability measures	C. Bertucci
10:00	Linear quadratic Mean Field Games in Hilbert spaces and some applications	D. Ghilli
10:45	Coffee Break	-
11:15	Regularity and quantitative stability for the Gibbs conditioning principle on path space via McKean-Vlasov control	G. Conforti
12:00	Major/minor MFG: common noise helps	F. Delarue
12:45	End of the talks	

Presentations abstracts

Martino Bardi (Università di Padova)

Title: Ergodic behaviour of deterministic Mean Field Games with non-monotone interactions.

Abstract: We consider deterministic Mean Field Games (MFG) in all Rn with a cost functional continuous with respect to the distribution of the agents and satisfying a gap condition at in nity. We rst show that from any equilibrium of the static MFG with such a cost one can build a solu- tion of the ergodic MFG system of 1st order PDEs with the same cost. Conversely, the measure component of any solution to the ergodic MFG must solve the associated static MFG. These properties hold under general conditions, including non-separable Hamiltonians and some models with congestion e ects. This leads to new existence and uniqueness results for deterministic ergodic MFG. Next we address the long-time limit of the solutions to nite horizon MFG with H(x; p;m) = jpj2 □ F(x;m) and cost functional F satisfying various additional assumptions. The parabolic system arising when the agents are a ected by independent non-degenerate noise was treated by Cardaliaguet, Lasry, Lions, and Porretta. For deterministic MFG the problem was studied by Cardaliaguet on the torus [1] and by Cannarsa et al. in the whole space [2], and both papers use in a crucial way a monotonicity condition on the cost functional. This condition strengthens the classical one by Lasry and Lions that implies a preference for less crowded areas and implies uniqueness of the solution to the MFG system. Our main assumption, instead, is about the set of minima of the cost F. It allows the aggregation of the agents and the existence of multiple solutions. We prove the convergence of the distribution of the agents and of the value function to a solution of the ergodic MFG system as the horizon of the game tends to in nity. Most of this research is joint work with Hicham Kouhkouh (University of Graz) and can be found in [3]. [1] P. Cardaliaguet: Long time average of rst order mean eld games and weak KAM theory, Dyn. Games Appl. 3 (2013), 473 - 488. [2] P. Cannarsa, W. Cheng, C. Mendico, K. Wang: Long-time behavior of rst-order mean eld games on Euclidean space, Dyn. Games Appl. 10 (2020), 361 - 390. [3] M. Bardi, H. Kouhkouh: Long-time behaviour of deterministic Mean Field Games with non-monotone interactions, SIAM J. Math. Anal. to appear.

Charles Bertucci (École Polytechnique, Paris)

Title: Miscellaneous on Hamilton Jacobi equations on the space of probability measures.

Abstract: In this talk, I will discuss i) comparison principles, ii) continuity of value functions, iii) singular terminal conditions, on two examples of HJ equations. The first one is somehow a standard one and arose in variations of optimal transport problems. The second one is the control of the so-called Dyson equation, which we studied with P.L. Lions and P.E. Souganidis. I will explain, on these two drastically different cases, some strategies of proof which seem somehow general for the three points at interest.

Fabio Camilli (Sapienza Università di Roma)

Title: On quasi-stationary Mean Field Games of Controls.

Abstract: In this talk, we will explore quasi-stationary Mean Field Games of Controls. In these problems, the strategy-choice mechanism of the agent differs from the classical one: the generic agent cannot predict the evolution of the population and instead chooses its strategy based solely on the information available at the current moment, without anticipating future developments. Furthermore, the dynamics of an individual agent is influenced not only by the distribution of agents but also by the distribution of their optimal strategies.
We demonstrate the existence and uniqueness of the solution for the corresponding quasi-stationary Mean Field Games system under various sets of hypotheses and provide examples of models that fall within these parameters.

Pierre Cardaliaguet (Université Paris Dauphine)

Title: A free boundary problem arising in mean field games

Abstract: In this joint work with Sebastian Munoz (Chicago) and Alessio Porretta (Rome Tor Vergata), we study the behavior of solutions to the first-order mean field game system with a local coupling, when the initial density is a compactly supported function on the real line. Our results show that the solution is smooth in regions where the density is strictly positive, and that the density itself is globally continuous. Additionally, the speed of propagation is determined by the behavior of the cost function near small values of the density. When the coupling is entropic, we demonstrate that the support of the density propagates with infinite speed. On the other hand, for a power-type coupling, we establish finite speed of propagation, leading to the formation of a free boundary. We prove that under a natural non-degeneracy assumption, the free boundary is strictly convex and enjoys $C^{1,1}$ regularity. We also establish sharp estimates on the speed of support propagation and the rate of long time decay for the density. Moreover, the density and the gradient of the value function are both shown to be H\"older continuous up to the free boundary. Our methods are based on the analysis of a new elliptic equation satisfied by the flow of optimal trajectories. The results also apply to mean field planning problems, characterizing the structure of minimizers of a class of optimal transport problems with congestion.

René Carmona (Princeton University)

Title: Optimal control of conditional processes: old and new.

Abstract: In this talk, we consider the conditional control problem introduced by P.L. Lions in his lectures at the College de France in November 2016. As originally stated, the problem does not fit in the usual categories of control problems considered in the literature, so its solution requires new ideas, if not new technology. In his lectures, Lions emphasized some of the major differences with the analysis of classical stochastic optimal control problems and in so doing, raised the question of the possible differences between the value functions resulting from optimization over the class of Markovian controls as opposed to the general family of open loop controls. While the equality of these values is accepted as a "folk theorem" in the classical theory of stochastic control, optimizing an objective function whose values strongly depend upon the past history of the controlled trajectories of the system is a strong argument in favor of differences between the optimization results over

these two different classes of control processes. The goal of the talk is to elucidate this quandary and provide elements of response to Lions' original conjecture, both in the case of "soft killing" (R.C. - Lauriere - Lions, Illinois Journal of Math) and in the case of hard killing (R.C. - Lacker, in progress).

Giovanni Conforti (Università di Padova)

Title: On the propagation of asymptotic convexity along HJB equations and applications

Abstract: It is well known that log-concavity is preserved along the heat flow. Equivalently, this can be phrased as the fact that if the terminal condition is convex, then the solution of the Hamilton-Jacobi Bellman (HJB) equation stays convex at all times. Quite surprisingly, this basic geometric invariance principle has found in recent and not-so-recent times several striking applications. For example, it was used back in 1976 by Brascamp and Lieb to establish convexity of the ground state for Schrödinger operators. Furthermore, over the last couple of years it has been employed to generalise Caffarelli's theorem on the existence of Lipschitz transport maps, to show the exponential convergence of scaling algorithms for entropic optimal transport such as Sinkhorn's algorithm, and as a tool in the renormalisation approach to functional inequalities. In this talk, I will show how some probabilistic constructions based on coupling ideas reveal the existence of families of functions that are invariant for Hamilton-Jacobi-equations but that are not necessarily convex. More precisely, they are only asymptotically convex in the sense that the integral of the second directional derivative over a long segment is non-negative. Applications to the convergence of Sinkhorn's algorithm and the turnpike phenomenon in mean field control will be discussed.

François Delarue (Université Côte d'Azur, Nice)

Title: Major/minor MFG: common noise helps.

Abstract: The purpose of the talk is to revisit conditions under which major/minor MFGs have a unique equilibrium over time intervals of arbitrary length. To this end, it is assumed that the costs to the minor players are monotone and that the intensity of the noise (referred to as `common') driving the dynamics of the major player is large enough with respect to some characteristic quantities of the game. These conditions allow us to construct a classical solution to the corresponding master equation.

Joint work with Chenchen Mou (City University of Hong-Kong)

Daria Ghilli (Università di Pavia)

Title: Linear quadratic Mean Field Games in Hilbert spaces and some applications.

Abstract: We study a class of linear quadratic Mean Field Games (MFG) in infinite dimension, where the state variable lives in a Hilbert space. Our motivations are problems where the state equation is a PDE or a delay equation which can be written as an ODE in a suitable Hilbert space. As a starting point, we study the case, considered in most finite dimensional contributions on the topic, where the dependence on the distribution enters just in the objective functional through the mean. This feature allows, similarly to the finite dimensional

case, to reduce the usual mean field game system to a Riccati equation and a forward-backward coupled system of abstract evolution equations. Such system is completely new in infinite dimension and no results have been proved on it so far. We show existence and uniqueness of solutions for such system, applying a delicate approximation procedure. We apply the results to a production output planning problem with delay in the control variable.

Diogo A. Gomes (KAUST Saudi Arabia)

Title: Weak-strong uniqueness for mean-field games.

Abstract: We address the uniqueness of stationary first-order Mean-Field Games (MFGs). Despite well-established existence results, establishing uniqueness, particularly for weaker solutions in the sense of monotone operators, remains an open challenge. Building upon the framework of monotonicity methods, we introduce a linearization method that enables us to prove a weak-strong uniqueness result
for stationary MFG systems. In particular, we give explicit conditions under which this uniqueness holds.

Fausto Gozzi (Luiss, Roma)

Title: On Mean Field Games and Mean Field Control in infinite Dimension .

Abstract: In this talk we report on some recent works with various coauthors.
On one side the work (with A. Cosso, F. Masiero, I. Kharroubi, H. Pham, M. Rosestolato) on the optimal control of McKean-Vlasov equations valued in Hilbert spaces. On the other side the work (with S. Federico, D. Ghilli, A. Swiech) on Mean Field Games in infinite dimension.
We start presenting some examples on both topics and on their connections.
On both topics, we discuss the results and we present some going on ideas and further work.

Jameson Graber (Baylor University)

Title: Remarks on Potential Mean Field Games.

Abstract: A potential game is one for which minimizers of a certain potential are Nash equilibria. Many examples of mean field games are also potential. In this talk, I will give an overview of potential games both from classical game theory and mean field games. A particularly important motivation is the "selection problem," that is, the search for criteria that uniquely select one Nash equilibrium when there are many. I will explore whether (and in what sense) minimizing the potential gives a satisfactory selection principle.

Ziad Kobeissi (Université Paris-Saclay)

Title: A mean-field-game approach to overfishing.

Abstract: In this talk, we investigate an instance of the tragedy of the commons in spatially distributed harvesting games. The model we choose is that of a fishes' population that is governed by a parabolic bistable equation and that fishermen harvest. We assume that, when no fisherman is present, the fishes' population is invading (mathematically, there is an invading travelling front). Is it possible that fishermen, when acting

selfishly, each in his or her own best interest, might lead to a reversal of the travelling wave and, consequently, to an extinction of the global fishes' population? To answer this question, we model the behaviour of individual fishermen using a Mean Field Game approach, and we show that the answer is yes. We then show that, at least in some cases, if the fishermen coordinated instead of acting selfishly, each of them could make more benefit, while still guaranteeing the survival of the fishes' population. Our study is illustrated by several numerical simulations

Paola Mannucci (Università di Padova)

Title: Deterministic Mean Field Games with non coercive Hamiltonian.

Abstract: I will talk on a research project about some models of evolutive deterministic MFGs where the Hamiltonian is not coercive in the gradient term because the dynamics of the generic player must fulfill some state constraints (holonomic or non-holonomic) or fail to be strongly controllable. First of all, I will outline the model where the generic player can move in the whole space, but it has some forbidden directions. As an example, I will consider the Heisenberg case. Afterwards, I will treat the case where the generic agent can control its state only through the acceleration. I consider both the case where the agents can move in the whole space and the case where the agents are also constrained to remain in a given bounded region. In particular I will explain in detail the case where the agents are constrained to remain in an interval. I will study the existence of weak solutions and their representation by means of relaxed equilibria in the Lagrangian interpretation which describes the game in terms of a probability measure on optimal trajectories. To get the existence of such equilibria, we consider the associated constrained optimal control problem and we obtain a closed graph result for the multivalued map which associates to an initial condition the set of the optimal trajectories starting from that point.

Alpar Meszaros (Durham University)

Title: Hidden monotonicity and canonical transformations for Mean Field Games and master equations.

Abstract: In this talk we propose some novel monotonicity conditions applicable for Mean Field Games through the exploration of finite dimensional canonical transformations. In classical Hamiltonian mechanics, canonical transformations are coordinate transformations on the phase space, which preserve the structure of Hamilton's equations. The proposed monotonicity conditions contribute to establishing new global well-posedness results for the associated master equations, also in the case of potentially degenerate idiosyncratic noise. Additionally, we show that recent advancements in global well-posedness results, specifically those related to displacement semi-monotone and anti-monotone data, can be easily obtained as a consequence of our main results. The talk will be based on joint works with Mohit Bansil (UCLA).

Sepideh Mirrahimi (CNRS, Université de Montpellier)

Title: Filling the gap between Hamilton-Jacobi equations and stochastic individual based models.

Abstract: An approach involving Hamilton-Jacobi equations has been developed during the last decades to study the long term eco-evolutionary dynamics of populations. How this approach is related to stochastic individual based models ? Can we interpret the results within this approach with regard to the corresponding individual based models ? In this talk I will present a work in collaboration with N. Champagant, S. Méléard and C. Tran, where we address this question. Our analysis leads to a correction of the Hamilton-Jacobi equations, keeping track of small sub-populations in large population approximations.

Emanuela Radici (Università dell'Aquila)

Title: Deterministic many particle limits for degenerate second order traffic models.

Abstract: We study the well-posedness and the deterministic many particle limits for second order Follow-the-Leader type models involving challenging nonlinearities which naturally appear in the modelling of crowd dynamics. We aim to validate the second-order particle approach for a class of traffic models characterised by the intrinsic mechanism that the reaction-time of the drivers depends on both inertial and congestion terms. We also consider the many particle limit in the vanishing inertia regime to recover the expected transport equation with nonlinear mobility. This is a joint work with D. Mazzoleni and F. Riva.

Luca Rossi (Sapienza Università di Roma)

Title: A MFG model of knowledge diffusion: convergence to a balanced growth path.

Abstract: In 2014, the economists R.E. Lucas and B. Mollproposed a mean-field game model to describe the dynamics of economic systems.In their model, production is based on one's technological knowledge, and the latter may grow by meeting people and learning from them. This leads to a MFG system where a backward Hamilton-Jacobi-Bellman equation is coupled with a forward KPP equation with nonlocal reaction term. In a first work with A. Porretta, we proved the existence of traveling waves, representing balanced growth paths for the described economy. In this talk, I will present a stability result for the balanced growth path, conjectured by the economists, obtained in collaborationwith A. Porretta and L. Ryzhik on a slightly modified model that takes into account a technological frontier in the learning process.

Giuseppe Savaré (Università Bocconi, Milano)

Title: Dissipative evolutions of probability measures.

Abstract: The study of Wasserstein gradient flows generated by displacement-convex functionals strongly relies on the metric-variational formulation of the implicit Euler scheme provided by the JKO/Minimizing Movement method.

The case of general dissipative evolutions (when the driving probability vector field is not generated by the Wasserstein gradient of some functional) appears more complicated. The theory presents two very different situations, depending on whether the probability vector field is dissipative along any coupling of measures (which we will call totally dissipative) or only along the optimal ones, that minimize the Wasserstein distance.

We will show that in the first case it is possible to construct a fully satisfying Lagrangian theory based on classical results in Hilbert spaces, which has interesting consequences also for the convergence of explicit schemes.

We will also show that a dissipative probability vector field defined everywhere and continuous is always totally dissipative. As a consequence we can see that in the Wasserstein setting it is impossible to approximate a dissipative (but not totally dissipative) probability vector field by continuous ones. This fact clarifies why the case of non-totally dissipative probability vector fields requires ad hoc investigation. (In collaboration with Giulia Cavagnari and Giacomo Sodini).

Benjamin Seeger (University of Texas Austin)

Title: Some well-posedness results for HJB equations on the space of probability measures

Abstract: In this talk, the well-posedness of HJB equations on the space of probability measures will be discussed. The focus will be on equations with idiosyncratic noise terms and nonconvex Hamiltonians. We present various methods for proving the comparison principle of viscosity solutions, as well as a new existence proof involving limits of equations on large, but finite, dimensional Euclidean space. Of particular importance for the latter is the ability to obtain uniform Lipschitz estimates with respect to certain weak metrics. This is joint work with S. Daudin and J. Jackson.

H. Mete Soner (Princeton University)

Title: Synchronization games

Abstract: Building on Winfree's work, the Kuramoto model (1975) has become the corner stone of mathematical models of collective synchronization, and has received attention in all natural sciences, engineering, and mathematics. These autonomous oscillators are coupled through a nonlinear interaction term which plays a central role in the long term behavior of the system. While the classical model postulates the dynamics of each oscillator in the form of a system of nonlinear ordinary differential equations, Yin, Mehta, Meyn, & Shanbhag (2010) use the mean-field game (MFG) formalism of Lasry & Lions. In this talk, in addition to the Yin et.al model, we also introduce a simpler two state model which can be seen as a discretization of the original one. We outline results showing that the mean field approach also delivers same type of results including the phase transition from incoherence to synchronization. In particular, in the discrete setting we provide a comprehensive characterization of stationary and dynamic equilibria along with their stability properties. In all models, while the system is unsynchronized when the coupling is not sufficiently strong, fascinatingly, they exhibit an abrupt transition to a full synchronization above a critical value of the interaction parameter. In the subcritical regime, the uniform distribution representing incoherence is the only stationary equilibrium. Above the critical interaction threshold, the uniform equilibrium becomes unstable and there is a multiplicity of stationary equilibria that are self-organizing. The discrete model with discounted cost present dynamic equilibria that spiral around the uniform distribution before converging to the self-organizing

equilibria. With an ergodic cost, however, unexpected periodic equilibria around the uniform distribution emerge.

This talk is based on joint works with Rene Carmona and Felix Hoefer of Princeton, and Quentin Cormier of INRIA.

Ariane Trescases (Université Paul Sabatier, Toulouse)

Title: Long-time behaviour of Keller-Segel type systems modeling local sensing

Abstract: We consider a class of cross-diffusion systems modeling chemotactic aggregation with local sensing. We will see that while reminiscent of the classical (minimal) Keller-Segel system, which may exhibit blow-up in finite time, this class of system typically possesses global-in-time solutions. Using entropy and duality methods, we discuss their rich long-time behaviour. We will also discuss possible extensions in presence of nonlinear self-diffusion.

Chiara Bernardini (Università di Trento)

Title: Some results on boundary value problems for Choquard equations

Abstract: We study the following nonlinear Choquard equation

$$-\Delta u+Vu = (I\alpha *|u|p)|u|p-2u \text{ in } \Omega \subset RN$$

where $N \geq 2$, $p \in (1, +\infty)$ and $V(x)$ is a continuous radial function such that $\inf x\in\Omega V > 0$.

First, assuming to have Neumann or Dirichlet boundary conditions, we prove existence of a positive radial solution when Ω is an annulus, or an exterior domain of the form $RN \setminus B\bar{a}(0)$. We also provide a nonexistence result: if $p \geq N+\alpha$ the corresponding Dirichlet problem does not have any nontrivial regular solution in strictly $N-2$ star-shaped domains.

Finally, when considering annular domains, letting $\alpha \to 0+$ we obtain an existence result for the corresponding local problem with power-type nonlinearity.

This talk is based on a joint work with A. Cesaroni.

Rosanna Capuani (University of Arizona)

Title: Introduction to first order MFG under state constraints

Abstract: In this talk, we give an introduction to deterministic mean-field games for which agents are confined in a closed domain with smooth boundary.

In this case, the existence of Nash equilibria cannot be deduced as for an unrestricted state space, because for a large set of initial conditions the uniqueness of solutions to the minimization problem solved by each agent is no longer guaranteed. Therefore, we attack the problem by considering a relaxed version of it, for which the existence of equilibria can be proved by set-valued fixed-point arguments. We will also show that this new solution definition satisfies a Mean Field Games system in an appropriate pointwise sense.

Samuel Daudin (Université Côte d'Azur, Nice)

Title: Stable Optimal Parameters of Mean-Field Neural ODEs

Abstract: Performing regression tasks with deep neural networks can be modeled as an optimal control problem for an ordinary differential equation. We investigate a relaxation of this problem where controls are taken to be probability measures over the parameter space and the cost involves an additional entropy penalization. We are particularly interested in the stability of the optimal parameters -- where stability is understood in terms of unique solvability of a certain linearized system. We show that, for « a lot » of initial data (in terms of the initial distribution of the features), there is actually a unique stable global minimizer in the control problem. Moreover we prove that the (continuous analog of the) gradient descent with backpropagation converges exponentially fast when initialized nearby a stable minimizer. This is a joint work with François Delarue.

Katharina Eichinger (École Polytechnique, Paris)

Title: On the turnpike property in mean field games

Abstract: In this talk we prove the exponential turnpike property for a class of mean field games on R^d. The exponential turnpike property states that optimal trajectories stay exponentially close to a stationary state, called turnpike, if they are far from the initial and final time.

Our technique is based on coupling by reflection adapted to controlled processes allowing us to treat controlled dynamics governed by an asymptotically convex potential. This enables us to prove existence and uniqueness of mean field game problems and their ergodic counterpart without monotonicity assumptions on the cost but rather a smallness condition on the dependence of the measure variable, and finally the exponential turnpike property. Based on joint work with Alekos Cecchin, Giovanni Conforti and Alain Durmus.

Joe Jackson (University of Chicago)

Title: From entropic propagation of chaos to exponential concentration bounds for stochastic particle systems

Abstract: In this talk, I am going to discuss an ongoing joint work with Antonios Zidritis, in which we point out a connection between two notions appearing in the asymptotic theory of weakly interacting particle systems. In particular, we show that "controlled entropic propagation of chaos" (a concept which will be introduced in the talk) implies exponential concentration bounds for the empirical measure of the system. This connection is useful, because controlled entropic propagation of chaos can be established for certain singular kernels by adapting an argument of Jabin and Wang, while (to the best of our knowledge) no exponential concentration bounds are currently available in this setting.

Luciano Marzufero (Libera Università di Bolzano)

Title: A single player and a mass of agents: a pursuit evasion-like game

Abstract: We study a finite-horizon differential game of pursuit-evasion like, between a single player and a mass of agents. The player and the mass directly control their own evolution, which for the mass is given by a first order PDE of transport equation type. Using also an adapted concept of non-anticipating strategies, we derive an infinite dimensional Isaacs equation, and by dynamic programming techniques we prove that the value function is the unique viscosity solution on a suitable invariant subset of a Hilbert space.

Elizabeth Matter (Baylor University)

Title: Cluster Formation in Iterated Mean Field Games

Abstract: We look at a simple first-order Mean Field Game that gives players incentive to congregate. With a short enough time horizon, this type of game has a unique Nash Equilibrium given an initial distribution of players. Since the game is played only for a short time, we consider iterating the game, each new iteration starting at the final distribution of the previous game. We prove that after sufficiently many iterations, the players do congregate in tighter and tighter clusters and show where these clusters form.

Nikifiros Mimikos-Stamatopoulos (University of Chicago)

Title: Sharp convergence rates for mean field control in the region of strong regularity.

Abstract: This is joint work with P. Cardaliaguet, J. Jackson, and P. E. Souganidis. We study the convergence problem for mean field control, also known as optimal control of McKean- Vlasov dynamics. We assume that the data is smooth but not convex, and thus the limiting value function $U : [0, T] \times P2(Rd) \to R$ may not be differentiable. In this setting, the first and last named authors recently identified an open and dense set O on which the limiting value function is C1, and solves the relevant infinite-dimensional Hamilton-Jacobi equation in a classical sense. In the present paper, we use these regularity results (and some non-trivial extensions of them) to derive sharp rates of convergence. In particular, we show that the value functions for the N-particle control problems converge towards U with a rate of 1/N, uniformly on subsets of O which are compact in the p-Wasserstein space for some $p > 2$. A similar result is also established at the level of the optimal feedback controls. Importantly, the rate 1/N is the optimal rate in this setting even if U is smooth, and the optimal global rate of convergence is known to be slower than 1/N. Thus our results show that the optimal rate of convergence is faster inside of O than it is outside. As a consequence of the convergence of optimal feedbacks, we obtain a concentration inequality for optimal trajectories of the N-particle problem started from i.i.d. initial conditions.

Davide Francesco Redaelli (Università di Padova)

Title: A priori estimates and large population limits for some nonsymmetric Nash systems with semimonotonicity.

Abstract: A priori estimates on the derivatives of the solution to the Nash system are classically expected to allow to pass to the limit for infinitely many players and get to the Master Equation by a compactness argument. Nevertheless, obtaining those estimates is considered a hard problem, which has so far discouraged such a

bottom-up approach to the study of this limit. I will illustrate how a semimonotonicity assumption is sufficient in order to obtain the estimates for a Nash system with quadratic Hamiltonian and then identify a large population limit in a generalised (nonsymmetric) Mean-Field framework. This is a joint work with Marco Cirant (Padua).

Michele Ricciardi (Luiss, Roma)

Title: Parabolic first-order Mean Field Games with Neumann boundary conditions

Abstract: The primary objective of this talk is to understand first-order, time-dependent mean-field games with Neumann boundary conditions, a question that re- mains under-explored in the literature. This matter is particularly relevant given the importance of boundary conditions in crowd models. In our model, the Neumann conditions result from players entering the domain Ω accord- ing to a prescribed current j, for instance, in a crowd entry scenario into an open-air concert or stadium. We formulate the model as a standard mean-field game coupling a Hamilton-Jacobi equation with a Fokker-Planck equation. Then, we introduce a relaxed variational problem and use Fenchel- Rockafellar duality to study the relation between these problems. Finally, we prove the existence and uniqueness of solutions for the system using vari- ational methods.

Riemann International RIEMM School of Mathematics

INdAM

Thanks for your participation

Speakers

Martino Bardi *(Università di Padova)*
Charles Bertucci *(École Polytechnique, Paris)*
Fabio Camilli *(Sapienza Università, Roma)*
Pierre Cardaliaguet *(Université Paris Dauphine)*
Renè Carmona *(Princeton University)*
Giovanni Conforti *(École Polytechnique, Paris)*
Francois Delarue *(Université Côte d'Azur, Nice)*
Daria Ghilli *(Università di Pavia)*
Diogo A. Gomes *(KAUST Saudi Arabia)*
Fausto Gozzi *(Luiss, Roma)*
Jameson Graber (Baylor University)
Ziad Kobeissi *(Université Paris-Saclay)*
Pierre-Louis Lions *(Collège de France, Paris)*
Paola Mannucci *(Università di Padova)*
Nader Masmoudi *(NYU Abu Dhabi)*
Alpar Meszaros *(Durham University)*
Sepideh Mirrahimi *(Université de Montpellier)*
Emanuela Radici *(Università dell'Aquila)*
Luca Rossi *(Sapienza Università, Roma)*
Giuseppe Savaré *(Università Bocconi, Milano)*
Ben Seeger *(University of Texas Austin)*
H. Mete Soner *(Princeton University)*
Panagiotis E. Souganidis *(University of Chicago)*
Ariane Trescases *(Université Paul Sabatier, Toulouse)*

Organizing and Scientific Committee

Annalisa Cesaroni *(Università di Padova)*
Marco Cirant *(Università di Padova)*
Alessio Porretta *(Università Roma Tor Vergata)*
Daniela Tonon *(Università di Padova)*

Università degli Studi di Milano Bicocca
Politecnico Milano
CRUL Comitato Regionale di coordinamento delle Università Lombarde
Regione Lombardia
Comune di Varese
Finanziato dall'Unione europea NextGenerationEU
Ministero dell'Università e della Ricerca
Italiadomani Piano Nazionale di Ripresa e Resilienza
Tor Vergata Università degli Studi di Roma